**Solution to Week #7’s (Q2) problem (11/28 – 12/2):**

1-Point Scenario Strategy:

**Plug in 10.**

One nice property of numbers in base 10 is that the digits can be written out in the form of a “polynomial”, with 10’s instead of x’s – for example, 2011 = 2\*103 + 0\*102 + 1\*101 + 1.

***Now, let’s capitalize on that!***

So, let’s say Dr. Bananaman comes up with a polynomial with 1-digit, positive integer coefficients, we tell him to plug in 10 for x, and he tells us f(10) = 1234.

We know the polynomial must have x3 as its highest exponent term, since having any 104’s in there would make f(10) greater than 10,000 (1\*104) and having no 103’s would stop it from getting any higher than 999 (9\*102+9\*101+9).

Then, let’s give the coefficients of f(x) variable names.

f(x) = ax3+bx2+cx+d.

f(10) = a(10)3+b(10)2+c(10)+d.

Now, we know that a(10)3, b(10)2, and c(10) will have a units digit of 0, since they are all multiples of 10. So, the only way for f(10) to end in 4 is if **the constant term (d) is 4**.

Likewise, a(10)3 and b(10)2 are multiples of 100, so they will not affect the tens digit of f(10). **That leaves c to be 3.**

Applying the same logic, **we get b to be 2 and a to be 1**. From f(10) alone, we get f(x). Solved!

***So, how do we make this a general strategy?***

1) Plug in 10

2) The units digit of the result is the constant term

3) The tens digit is the x coefficient

4) The hundreds digit is the x2 coefficient

5) …

1072358) You have the polynomial!

2-Point Scenario Strategy:

**Plug in 1, then plug in a power of ten greater than f(1).**

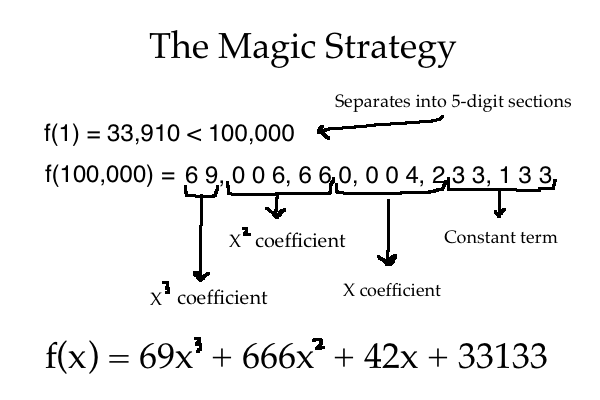
This is the really tricky part. If the coefficients of the secret polynomial can be more than one-digit long, we can’t apply the strategy we used in the 1-Point Scenario right off the bat.

Take the f(10) = 1234 example from before. If the coefficients must be positive integers, the only possible polynomial that will spit this out is f(x) = x3 + 2x2 + 3x + 4.

However, if they can have multiple digits, for all we know, f(x) could just be 1234 – the constant term isn’t confined to the units digit of f(10).

In order to prevent overlaps like that, we need to find a power of 10 larger than any of the coefficients, and use that instead of 10.

To find it: since the coefficients are all positive, the sum of the coefficients (which we can find using f(1) – check it out if you don’t see it at first) must be greater than each of the individual ones. So, if we plug in 1, we know what power of 10 we can plug in to “space the coefficients” in a way that they don’t overlap and conflict. Check it out in action below, for the secret polynomial 69x3 + 666x2 + 42x + 33133!

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*Correct answers on the 1-Point Scenario from:*

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Congrats to the above, thanks to everyone else that gave it a try, and happy studying for midterms! ☺